Why 'All Logic' can't be

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(maybe this should have started with the end, i.e. the PS)

""My propositions serve as elucidations in the following way: anyone who understands me eventually recognizes them as nonsensical, when he has used them—as steps—to climb up beyond them. (He must, so to speak, throw away the ladder after he has climbed up it.) He must transcend these propositions, and then he will see the world aright."'

— Wittgenstein, *Tractatus* 6.54

The Basic Setup

- For any natural number n, let L_n be a logic system of order n
- Let P_n be **all** the semantic and syntactic components (predicates, constants, variables, quantifiers, models etc) available in L_n
- Let $\mathcal{P}$ represent **every possible semantic and syntactic components** across all systems: $\mathcal{P} = P_1 \cup P_2 \cup P_3 \cup \cdots$

The Core Problem

- In any system L_n, we can *use* its "tools" P_n freely
- But we *can't formally talk about* P_n as a complete package within L_n itself

The Fatal Loop

Now consider $\mathcal{P}$ - the hypothetical collection of *all* semantic and syntactic components from *all* systems. To claim "$\mathcal{P}$ exists":

1. You'd need to make this claim in some specific system L_k
2. But $\mathcal{P}$ by definition includes tools from L_{k+1}, L_{k+2}, etc.
3. These tools aren't available in your starting system L_k

4. Therefore, L_k can't actually formalize the full $\mathcal{P}$ it's trying to describe

This amounts to the conclusion that there is no formal language in which I can say "$\mathcal{P}$ exists".

Conclusion

The very act of trying to say "All logic exists" (if by a "logic" we understand the totality of its semantic and syntactic components, regardless of its proof-theoretic and model-theoretic entailment) in any particular logical system undermines itself. We're left with:

- Particular logics we can work with (L_1, L_2, etc.)
- Grand claims about "all logic" that forever escape formalization

PS: This was a short text written on the margin of the Ontological Argument. It reveals why the entire tradition of ontological arguments—from Anselm's Proslogion to contemporary modal versions—commits a fundamental category error. If God represents ultimate reality or the source of all possible being, then God functions like our set P: the totality that encompasses all formal systems. But then any attempt to prove God's existence within a particular logical framework L faces the structural impossibility we have demonstrated—the system cannot formalize claims about what transcends its own boundaries. For philosophers, this provides a precise, formal reason why centuries of increasingly sophisticated ontological arguments have failed to compel: the enterprise itself is structurally incoherent. For believers, this offers unexpected liberation—faith need not await logical vindication because the divine necessarily exceeds the grasp of human formal systems. Rather than viewing this as a defeat for theism, it preserves what many traditions have long insisted: that God belongs to the realm of encounter and commitment, not demonstration and proof. The mystery remains mysterious not due to our current limitations, but due to the very structure of formal discourse itself.